MT/D11 : 8907

MTEC-1.3 : Basics of State Variable Techniques

Time : Three Hours Maximum Marks : 60

Note:- Attempt any five questions in all, selecting at - least one question from each unit.

UNIT-I

Q.1 a) Calculate eigen values and eigen vectors for the system given by the state matrix 06

$$A = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

b) A system is represented by the state matrix 06

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & -7 & -9 & -5 \end{bmatrix}$$

Calculate the modal matrix & Jordan Matrix for the system.

Q.2 a) Let A be nxn matrix. Using Caley Hamilton theorem show that A^K with $K \geq n$, can be written as a Linear combination of $\{I, A, \ldots, A^{n-1}\}$.

What modifications are necessary if the degree of minimal no of A is known to be m? 06

b) Find the eigen values, eigen vectors & Jordan form representation for the following matrices

$$(i)\ \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix} \qquad (ii)\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 4 & -4 & -3 & 4 \end{bmatrix}$$ 06

UNIT - II

Q.3 a) Explain with example(s) controllability & observability. 06

b) Convert the following state models into Jordan Canonical form and therefore comment on controllability & observability. 06

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} u(t) \quad (i)$$

$$y(t) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix} x(t) \quad (ii)$$

Q.4 a) An system is described by the state equations.

$$x(k+1) = F x(k) + G u(k) \quad (i)$$
$$y(k) = C x(k) + D u(k) \quad (ii)$$ 06

Where F, G, C & D are, respectively nxn, nxp, qxn and qxq scalar constant matrix and

K=0,1,2..........,Prove that the system is controllable if and only if the rank of (nxnp) controllability matrix U is n, i.e p(U)=n.

b) Obtain Jordan Canonical form realization of transfer function. 06

$$\hat{h}(z) = \frac{\hat{y}(z)}{\hat{u}(z)} = \frac{z+6}{z^3+5z^2+7z+3}$$

UNIT - III

Q.5 a) For scalar valued functions explain with suitable examples (i) Positive Definiteness (ii) Negative Definiteness (iii) Semi Definiteness. 06

b) Evaluate pulse response matrix for an n dimensional linear time in-variant single input/single output system describe by the state model 06

$$x(k+1) = F x(k) + g u(k) \quad (i)$$
$$y(k) = C x(k) + d u(k) \quad (ii)$$

Where F is nxn real constant matrix, g&c are respectively nx1 real constant vectors and d is constant scalar, K=0, 1, 2-----------------. 06

Q.6 a) Explain Lyapunov's stability theorem. 06

b) Applying Lyapunov's stability theorem comment on the stability of the system described by the state equations.

$$x_1(k+1) = -1.5\, x_1(k) \quad (i)$$
$$x_2(k+1) = -0.5\, x_2(k) \quad (ii)$$ 06

UNIT - IV

Q.7 a) Determine stability with the help of Jury's tabulation for the system having characteristic equation. 06

$$Z^3 + 3.3Z^2 + 4z + 0.8 = 0$$

b) Explain Lyapunov function, stability theorem of Lyapunov and instability theorem of Lyapunov. 06

Q.8 Write short notes on any Two of the following. 12

i) Caley Hamilton Theorem.

ii) Transfer function.

iii) Singular value Decomposition.